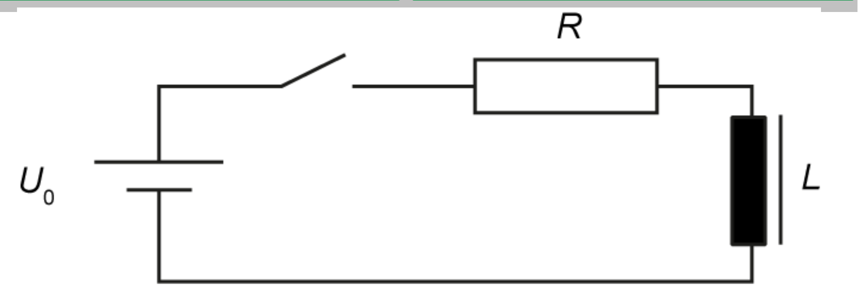
4. Elektrodynamik - zeitabhängige Ströme

4.1 Schaltvorgänge

4.2 Wechselstromkreise

4.3 Elektromagnetische Schwingungen und Wellen

RL-Schaltung



Maschenregel: $$U_0 - I \cdot R - L\frac{\mathrm{d}I}{\mathrm{d}t} = 0$$

Lösungsverfahren:
Trennung
der Variablen

$$L\frac{\mathrm{d}I}{\mathrm{d}t} = (U_0 - IR);$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{1}{L}(U_0 - IR);$$

$$\frac{dI}{dt} = \frac{R}{L}\left(\frac{U_0}{R} - I\right)$$

$$\frac{\mathrm{d}I}{\frac{u_0}{R} - I} = \frac{R}{L}\mathrm{d}t;$$

$$\int_0^I \frac{\mathrm{d}I^*}{\frac{u_0}{R} - I^*} = \int_0^t \frac{R}{L}\mathrm{d}t^*$$

$$\left[\ln\left(\frac{U_0}{R} - I\right) - \ln\left(\frac{U_0}{R}\right)\right] = -\frac{R}{L}\cdot t;$$

$$\ln\left(\frac{\frac{U_0}{R} - I}{\frac{U_0}{R}}\right) = -\frac{R}{L}\cdot t;$$

$$\frac{\frac{U_0}{R} - I}{\frac{U_0}{R}} = e^{-\frac{R}{L}\cdot t};$$

$$\frac{U_0}{R} - I = \frac{U_0}{R}\cdot e^{-\frac{R}{L}\cdot t};$$

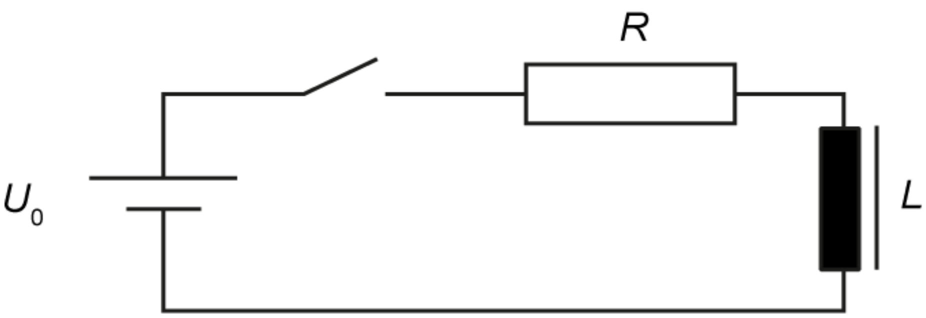
$$\boldsymbol{I = \frac{U_0}{R}\cdot\left(1 - e^{-\frac{R}{L}\cdot t}\right);}$$

RL-Schaltung

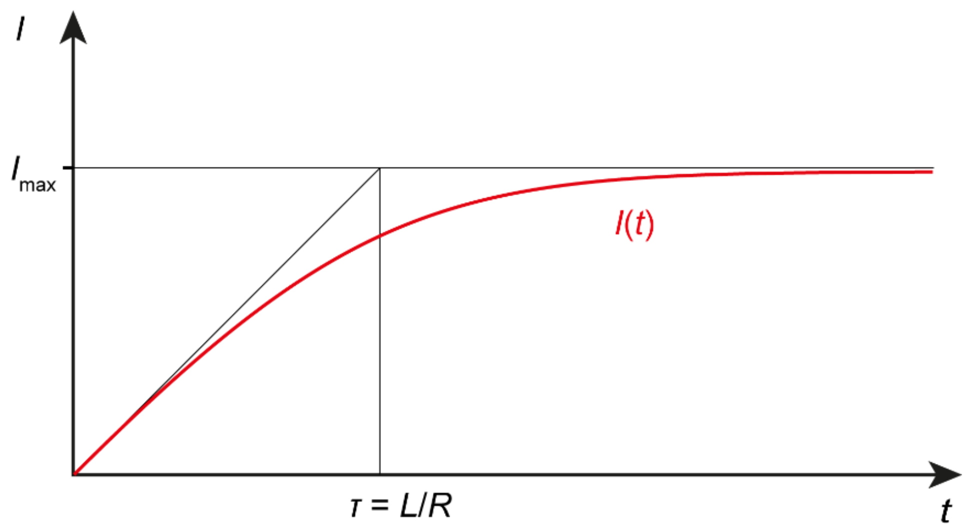
$$I = \frac{U_0}{R} \cdot \left(1 - e^{-\frac{R}{L} \cdot t}\right);$$

$\frac{U_0}{R}$: $I_{\max}$

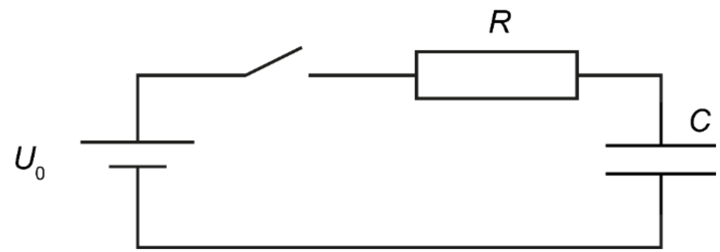
$\frac{R}{L}$: $\frac{L}{R} = \tau$ „Zeitkonstante“



$$I = I_{max} \cdot \left(1 - e^{-\frac{t}{\tau}}\right);$$



RC-Kreis



Maschenregel:

$$U_0 - I \cdot R - \frac{Q}{C} = 0\ ;$$

$$U_0 - \frac{\mathrm{d}Q}{\mathrm{d}t}R - \frac{Q}{C} = 0\ ;$$

Lösungsverfahren:
Trennung
der Variablen

$$RC \cdot \frac{dQ}{dt} = CU_0 - Q\ ;$$

$$\frac{dQ^*}{CU_0 - Q^*} = \frac{1}{RC}dt^*$$

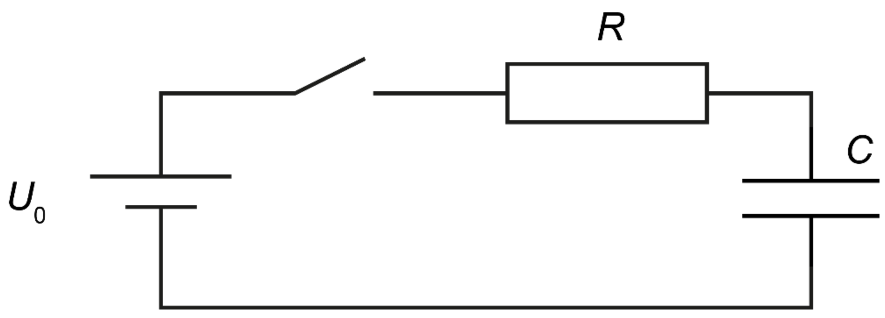
$$\int_0^Q \frac{dQ^*}{CU_0 - Q^*} = \int_0^t \frac{1}{RC}dt^*$$

$$\ln\frac{CU_0 - Q}{Cu_0} = -\frac{1}{RC} \cdot t$$

$$CU_0 - Q = CU_0 \cdot e^{-\frac{1}{RC}\cdot t}$$

$$\boxed{Q = \underbrace{CU_0}_{Q_{max}} \cdot \left(1 - \mathrm{e}^{-\frac{1}{RC}\cdot t}\right)}$$

RC-Kreis

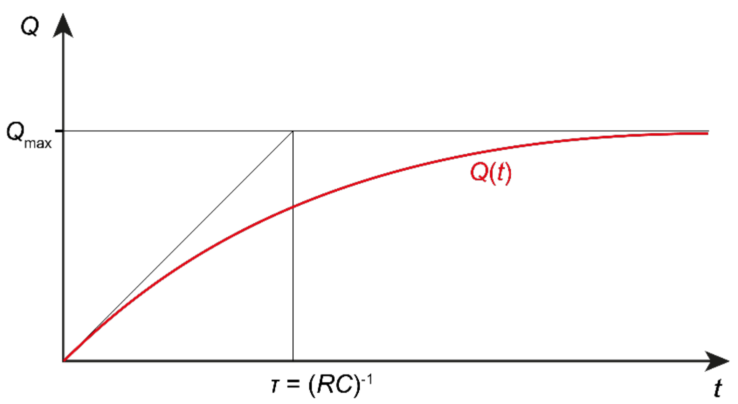


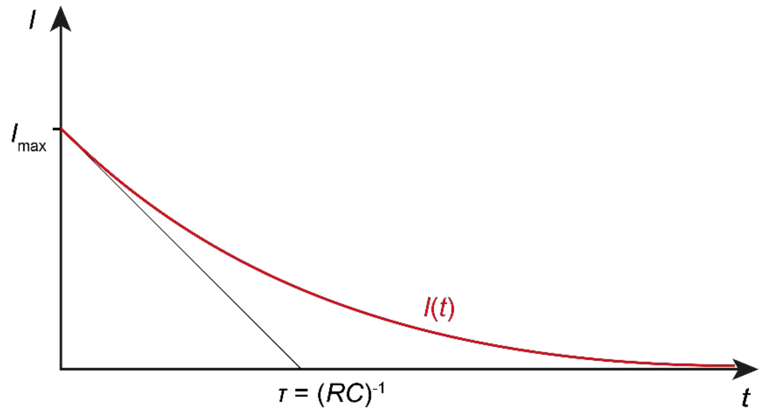
$$Q = CU_0 \cdot \left(1 - \mathrm{e}^{-\frac{1}{RC}\cdot t}\right)$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = -CU_0 \cdot \mathrm{e}^{-\frac{1}{RC}\cdot t} \cdot \left(-\frac{1}{RC}\right)$$

$$I = \underbrace{\frac{U_0}{R}}_{I_0} \cdot e^{-\frac{1}{RC}\cdot t}$$

$\frac{1}{RC} = \tau$ „Zeitkonstante“





Netzfrequenzen

Europa:	**50 Hz**
USA:	**60 Hz**
Dbahn:	**16 2/3 Hz**

Wechselspannungen gut transformierbar

2 Beschreibungen zur Spannung

Eulersche Formel:
$e^{i\omega t} = \cos\omega t + \mathrm{i}\cdot\sin\omega t\,;$

a) real

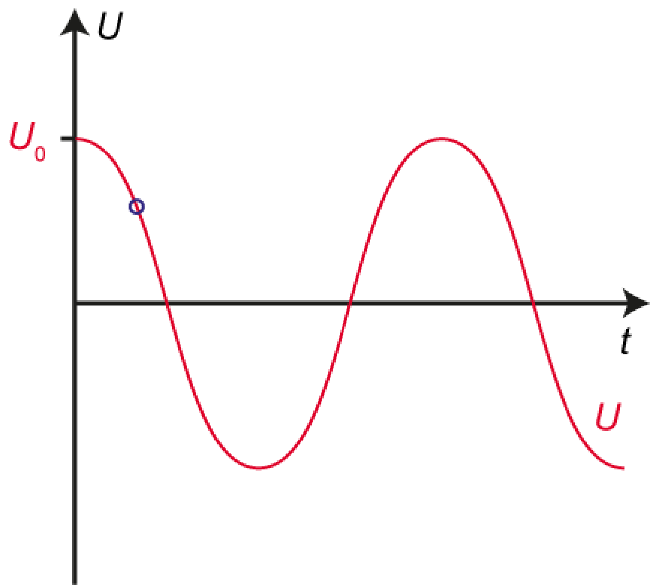
$$U = U_0 \cdot \cos\omega t \;;$$

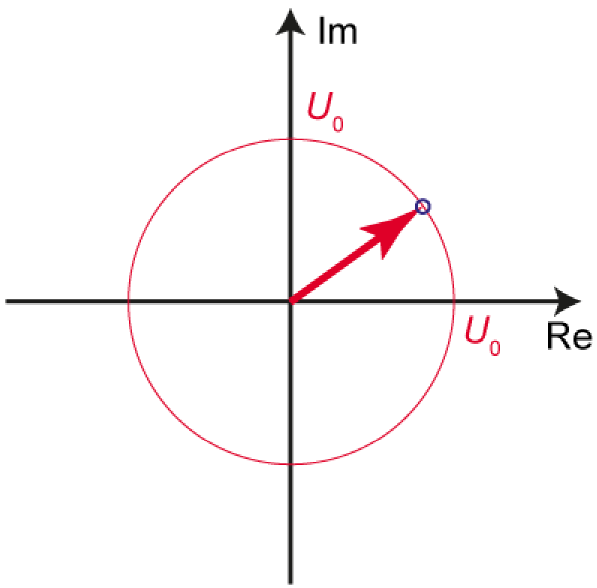
b) komplex

$$U = U_0 \cdot e^{i\omega t}\,;$$

$$U = U_0 \cdot \cos\omega t + \mathrm{i}\cdot U_0 \cdot \sin\omega t\,;$$

Funktions-graph





Zeiger-diagramm

2 Beschreibungen zur Spannung

Eulersche Formel:
$e^{i\omega t} = \cos\omega t + \mathrm{i} \cdot \sin\omega t\,;$

a) real

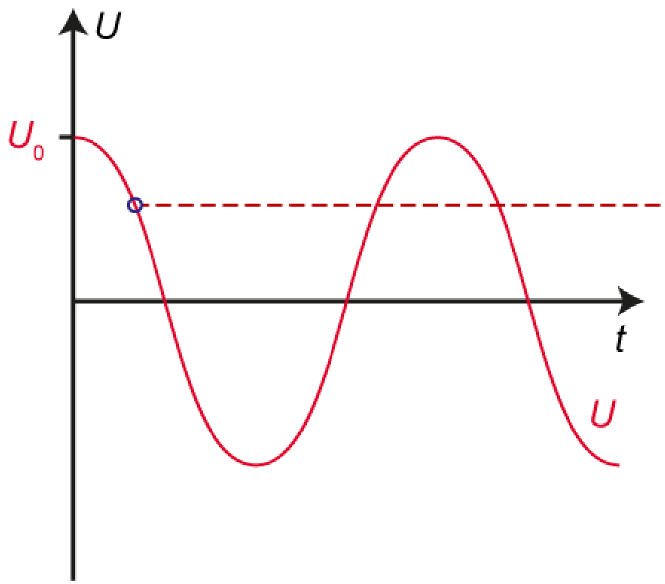
$$U = U_0 \cdot \cos\omega t\ ;$$

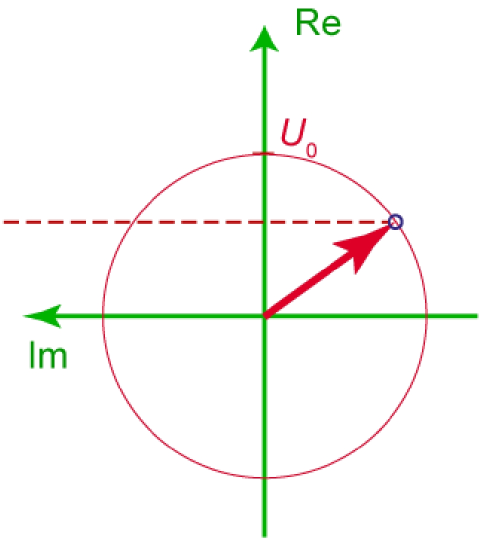
b) komplex

$$U = U_0 \cdot e^{i\omega t}\,;$$

$$U = U_0 \cdot \cos\omega t + \mathrm{i} \cdot U_0 \cdot \sin\omega t\,;$$

Funktions-graph

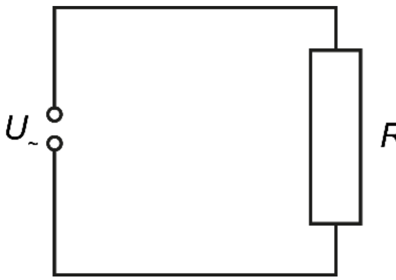




Zeiger-diagramm

gedreht

Wechselstromkreis mit Ohm‘schem Widerstand



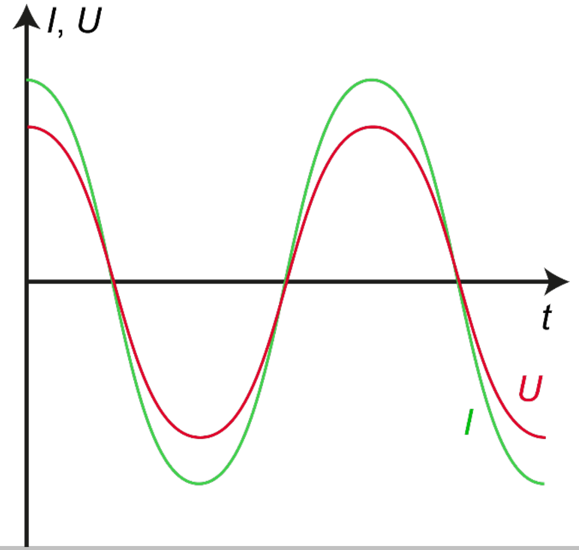
Maschenregel: $U_0 - I \cdot R = 0\,;$

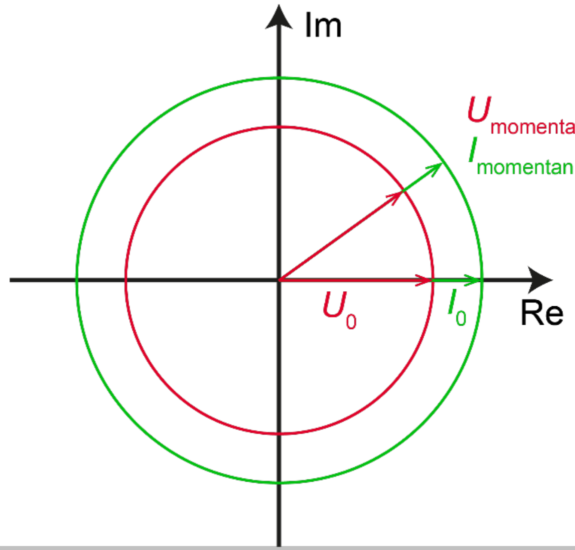
$$U_0 \cdot \cos \omega t - I \cdot R = 0\,;$$

$$I = \underbrace{\frac{U_0}{R}}_{I_0} \cdot \cos \omega t$$

$$U_0 \cdot e^{i\omega t} - I \cdot R = 0\,;$$

$$I = I_0 \cdot \mathrm{e}^{\mathrm{i}\omega t}$$





Leistung am Ohm'schen Widerstand

$$P(t) = U \cdot I = U_0 \cdot \cos(\omega t) \cdot I_0 \cdot \cos(\omega t)$$

$$P(t) = U_0 \cdot I_0 \cdot \cos^2(\omega t)$$

Im Mittel:

$$\bar{P} = \frac{1}{T} \underbrace{\int_0^T \cos^2(\omega t)\,\mathrm{d}t}_{\frac{\pi}{\omega} = \frac{\pi}{2\pi} T} \cdot U_0 \cdot I_0 \cdot$$

$$\bar{P} = \frac{1}{T} \cdot \frac{\pi}{2\pi} T \cdot U_0 \cdot I_0$$

$$\boxed{\bar{P} = \frac{1}{2} \cdot U_0 \cdot I_0 = \frac{1}{2} I_0^2 \cdot R = \frac{1}{2} \frac{U_0^2}{R}}$$

Leistung am Ohm‘schen Widerstand

$$\bar{P} = \frac{1}{2} \cdot U_0 \cdot I_0 = \frac{1}{2} I_0^2 \cdot R = \frac{1}{2} \frac{U_0^2}{R} \; ;$$

$$I_{eff} = \frac{I_0}{\sqrt{2}} \; ; \qquad U_{eff} = \frac{U_0}{\sqrt{2}} \; ;$$

Der Effektivwert einer Wechselspannung / eines Wechselstroms gibt den Wert an,
bei dem eine Gleichspannung/ein Gleichstrom
die gleiche Leitung an einem ohmschen Widerstand abgehen würde.

Ohmscher Widerstand: *R*

$$R = \frac{U_0}{I_0} = \frac{U_{eff}}{I_{eff}}$$

Komplexe Widerstände und Impedanzen

Def. : **Impedanz / Scheinwiderstand**

$$z = \frac{U_0}{I_0} = \frac{U_{eff}}{I_{eff}} \;;$$

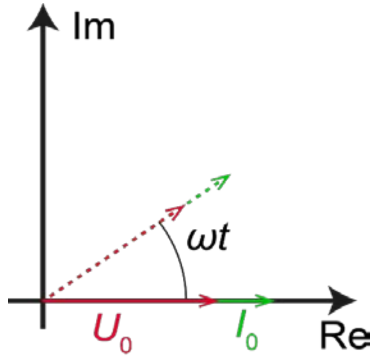
Komplexer Widerstand

$$\mathfrak{z} = \frac{U_0 \cdot e^{i\omega t}}{I_0 \cdot e^{i(\omega t + \varphi)}} \;;$$

$$\mathfrak{z} = \frac{U_0}{I_0} \cdot e^{i\varphi} = z \cdot e^{i\varphi} \;;$$

Beispiel: rein Ohm'scher Widerstand

$$z = \frac{U_0}{I_0} = \frac{U_{eff}}{I_{eff}} = R \;;$$



$$\mathfrak{z} = \frac{U_0 \cdot e^{i\omega t}}{I_0 \cdot e^{i(\omega t + \varphi)}} = \frac{U_0}{I_0} = R \;;$$

4.2.3 Induktiver Widerstand

Wechselstromkreis mit *L*

Wechselstromkreis mit *L*

$$U_0 \cdot \cos(\omega\, t) - L\,\frac{\mathrm{d}I}{\mathrm{d}t} = 0\;;$$

$$\mathrm{d}I = \frac{U_0}{L}\cdot\cos(\omega\, t)\cdot\mathrm{d}t\;;$$

$$I = \frac{U_0}{\omega L}\cdot\sin(\omega\, t)\;;$$

$$I = I_0\cdot\cos\left(\omega\, t - \frac{\pi}{2}\right)\;;$$

$$U_0 \cdot e^{i\omega\, t} - L\,\frac{\mathrm{d}I}{\mathrm{d}t} = 0\;;$$

$$\mathrm{d}I = \frac{U_0}{L}\cdot e^{i\omega\, t}\cdot\mathrm{d}t\;;$$

$$I = \frac{U_0}{i\omega L}\cdot e^{i\omega\, t}\;;$$

$$I = \frac{U_0}{i(-i)\omega L}\cdot(-i)\; e^{i\omega\, t}\;;$$

$$I = \frac{U_0}{\omega L}\cdot\; e^{i(\omega\, t - \frac{\pi}{2})}\;;$$

Der Strom „läuft“ der Spannung hinterher.

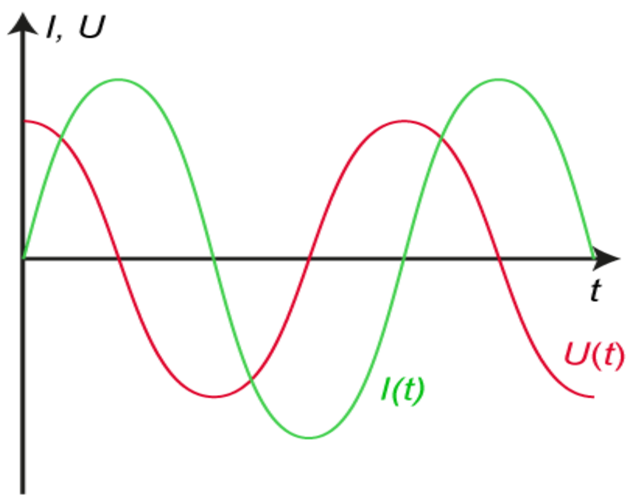
Ergänzung: $-\mathrm{i} = \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}\;;$

denn: $\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}} = \cos\left(-\frac{\pi}{2}\right) + \mathrm{i}\sin\left(-\frac{\pi}{2}\right)\;;$

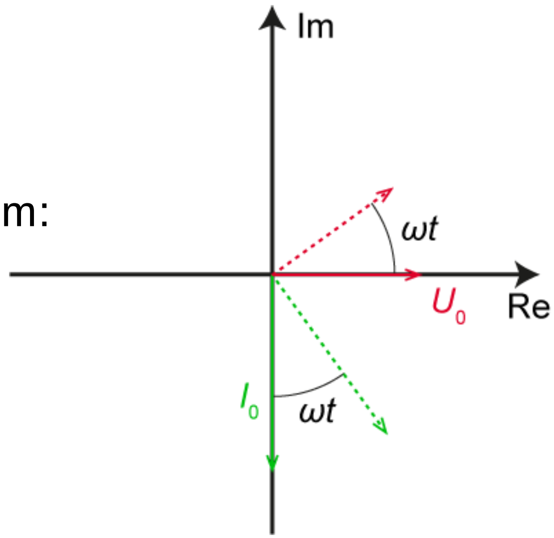
Wechselstromkreis mit *L*

$$I_0 = \frac{U_0}{\omega L} \;;$$

$$I = \frac{U_0}{i\omega L} \cdot e^{i\omega\, t} \;;$$



im Zeiger-diagramm:

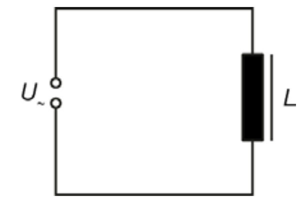


$$z_L = \frac{U_0}{I_0} = \omega\ L \;;$$

$$\phi_L = -\frac{\pi}{2} \;;$$

$$\zeta_L = \frac{U}{I} = i\omega\ L \;;$$

Leistung am induktiven Widerstand



$$P(t) = U_0 \cos \omega t \cdot I_0 \sin \omega t$$

mittlere Leistung:

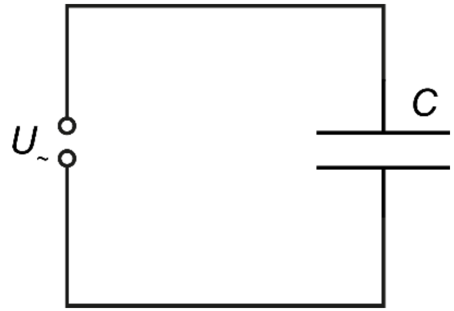
$$\bar{P} = \frac{1}{T} \int_0^T U(t) \cdot I(t)\, \mathrm{d}t \ ;$$

$$\bar{P} = \frac{1}{T} U_0 \cdot I_0 \cdot \int_0^T \cos\omega t \cdot \sin\omega t \, \mathrm{d}t \ ;$$

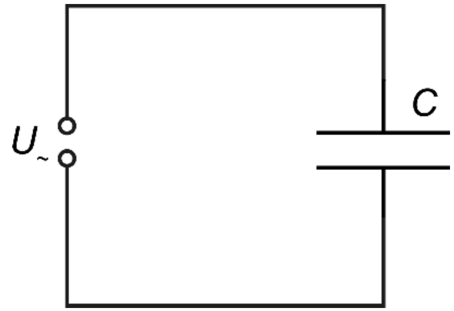
$$\boxed{\bar{P} = 0 \ ;}$$

4.2.4 Kapazitive Widerstände

Wechselstromkreis mit *C*



Wechselstromkreis mit *C*



$$U_0 \cos \omega t - \frac{q}{C} = 0\;;$$

$$U_0 \mathrm{e}^{\mathrm{i}\omega t} - \frac{q}{C} = 0\;;$$

$$q = C \cdot U_0 \cdot \cos \omega t\;;$$

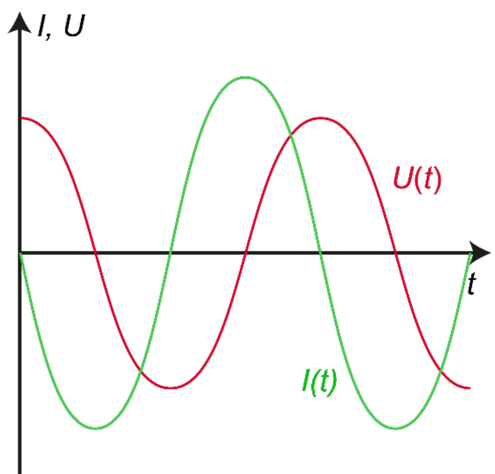
$$I = \frac{\mathrm{d}q}{\mathrm{d}t} = -\omega \cdot C \cdot U_0 \cdot \sin \omega t$$

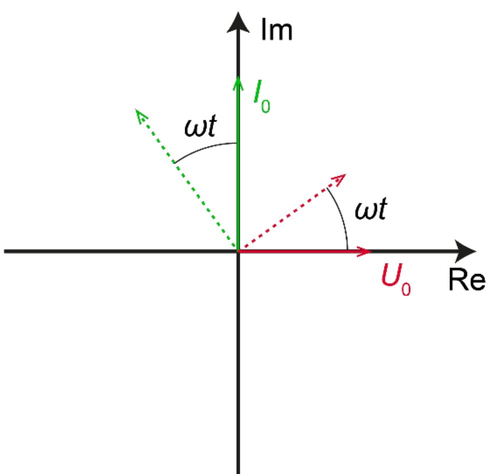
$$q = U_0 \cdot C \cdot \mathrm{e}^{\mathrm{i}\omega t}$$

$$I = \underbrace{\omega C U_0}_{I_0} \cdot \cos\left(\omega t + \underbrace{\frac{\pi}{2}}_{\varphi_c}\right)\;;$$

$$I = \mathrm{i}\omega C \cdot U_0 \cdot \mathrm{e}^{\mathrm{i}\omega t}\;;$$

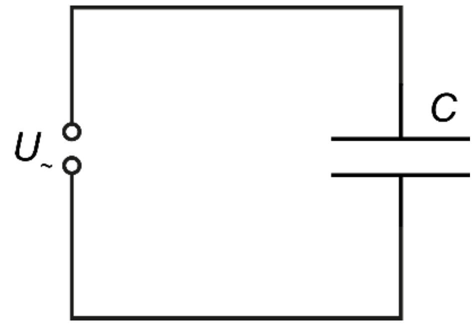
"Strom eilt der Spannung voraus"





Wechselstromkreis mit *C*

$$I_0 = \omega C \cdot U_0;$$



$$I = \mathrm{i}\omega C \cdot U\,;$$

Impedanz:

$$z_C = \frac{U_0}{I_0} = \frac{U_{eff}}{I_{eff}} = \frac{1}{\omega C}\;;$$

$$\varphi_c = \frac{\pi}{2}\;;$$

Komplexer Widerstand:

$$\zeta = \frac{U}{I} = \frac{1}{\mathrm{i}\omega C} = -\frac{\mathrm{i}}{\omega C}\;;$$

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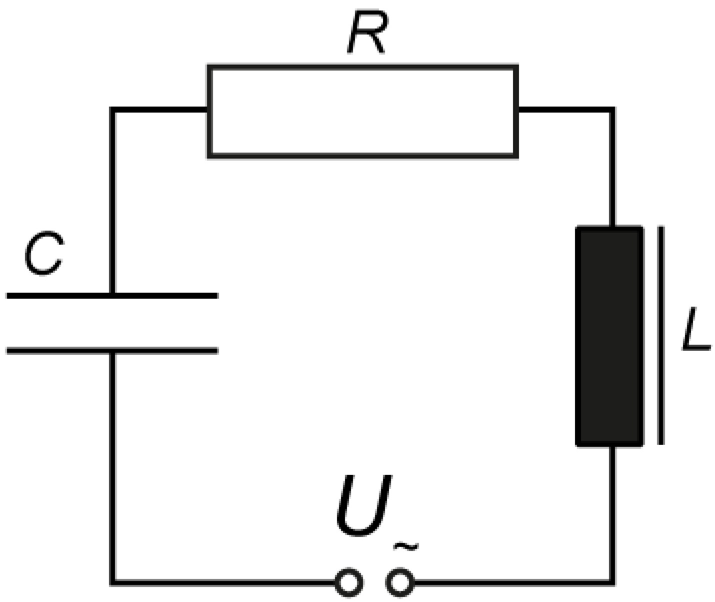
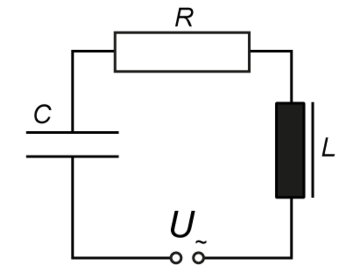
auch hier: $\boxed{\overline{P} = 0}$ (Blindleistung)

R-L-C **Serienkreis**



R-L-C Serienkreis

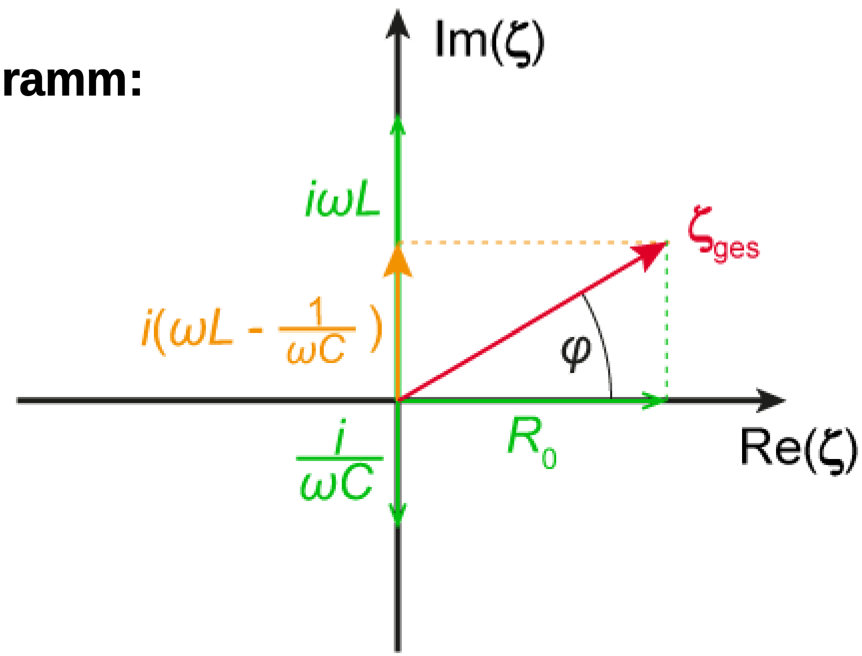


Darstellung im Zeigerdiagramm:

$$\zeta_R = R \ ;$$

$$\zeta_L = i\omega L \ ;$$

$$\zeta_R = -i\,\frac{1}{\omega C} \ ;$$



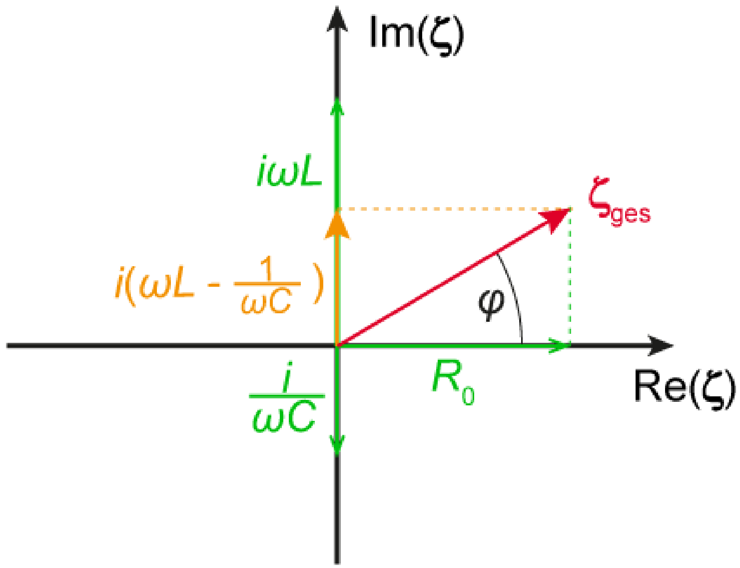
Scheinwiderstand:

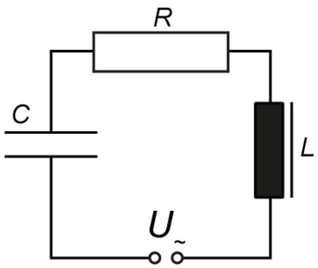
$$z = |\zeta| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2} \ ;$$

R^2: Wirk-widerstand; $\left(\omega L - \frac{1}{\omega c}\right)$: Blind-widerstand

$$|\zeta| = \sqrt{\mathrm{Re}(\zeta)^2 + \mathrm{Im}(\zeta)^2} \ ;$$

R-L-C **Serienkreis**



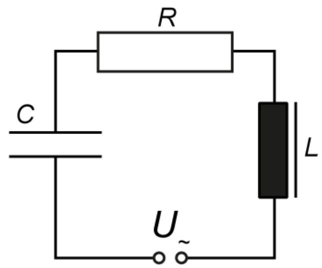


Phasenverschiebung:

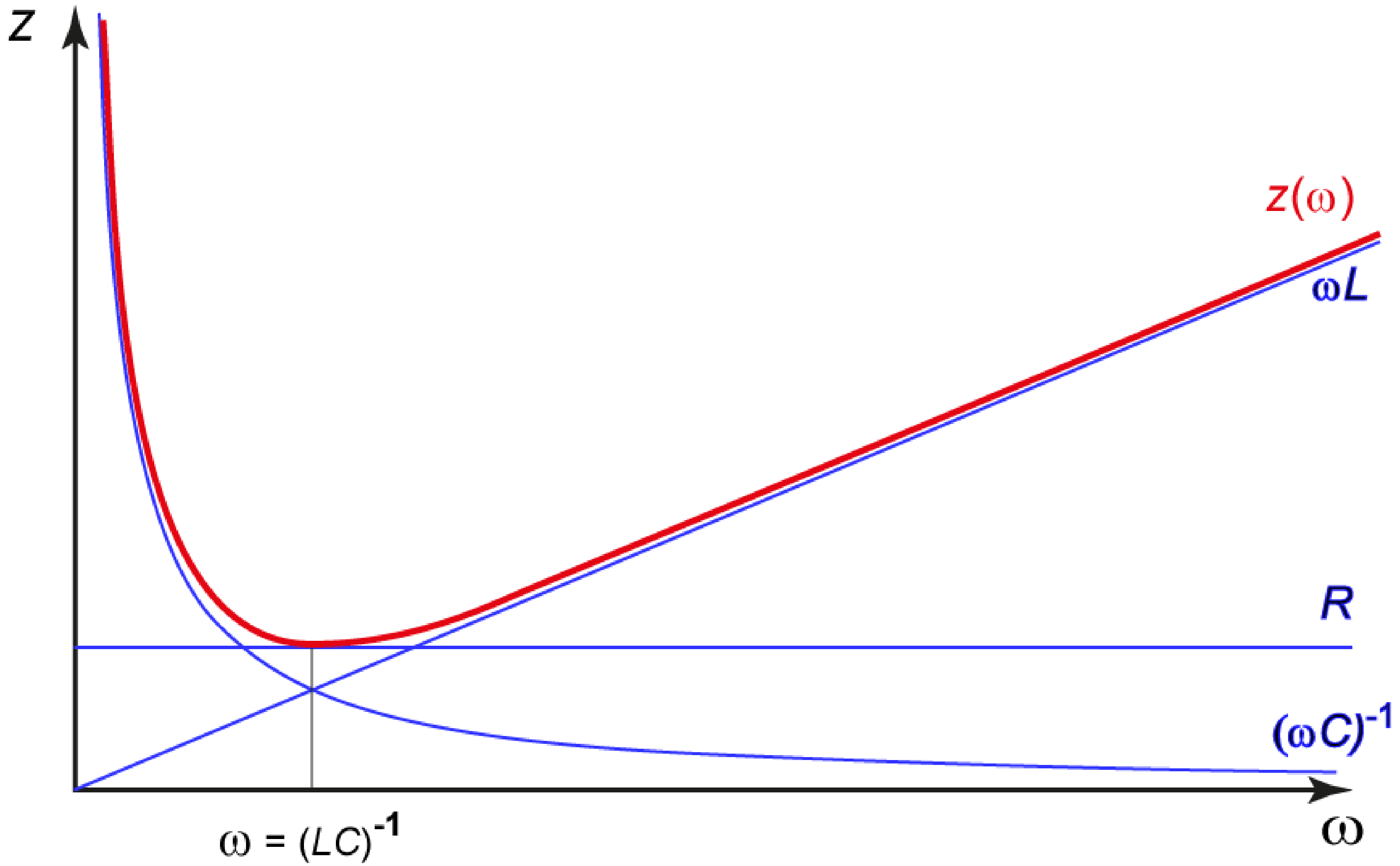
$$\tan\varphi = \frac{\mathrm{Im}(\zeta)}{\mathrm{Re}(\zeta)} = \frac{z_L - z_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R};$$

$$U = U_0 \cdot \cos\omega t \qquad I = I_0 \cos(\omega t - \varphi)$$

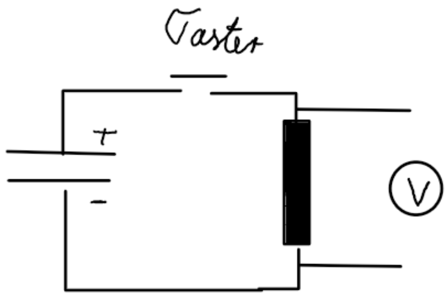
R-L-C Serienkreis



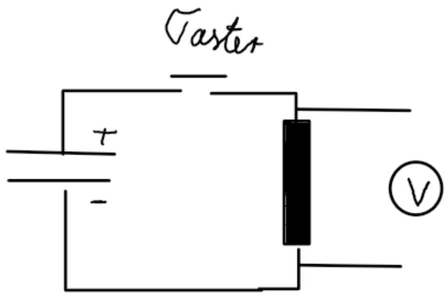
Der Scheinwiderstand z ist frequenzabhängig:



4.4.1 Einfacher LC-Schwingkreis



4.4.1 Einfacher LC-Schwingkreis



Maschenregel: $U_L + U_C = 0$;

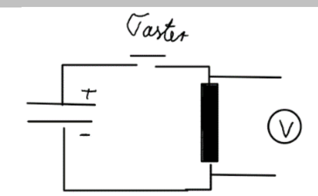
$$L \frac{dI}{dt} + \frac{q}{C} = 0$$

$$-L \frac{dI}{dt} = \frac{q}{C};$$

$$L \ddot{q} + \frac{1}{C} q = 0$$

{wie: $m\ddot{x} + kx = 0$; Schwingungs-DGL}

4.4.1 Einfacher LC-Schwingkreis



$$L\ddot{q} + \frac{1}{C}q = 0$$

Lösung:

$$q = q_m \cos(\omega t - \varphi)$$

mit

$$\omega = \frac{1}{\sqrt{LC}}$$

$$T = 2\pi\sqrt{LC}$$

Thomson - Gleichung

Verifizieren durch Einsetzen:

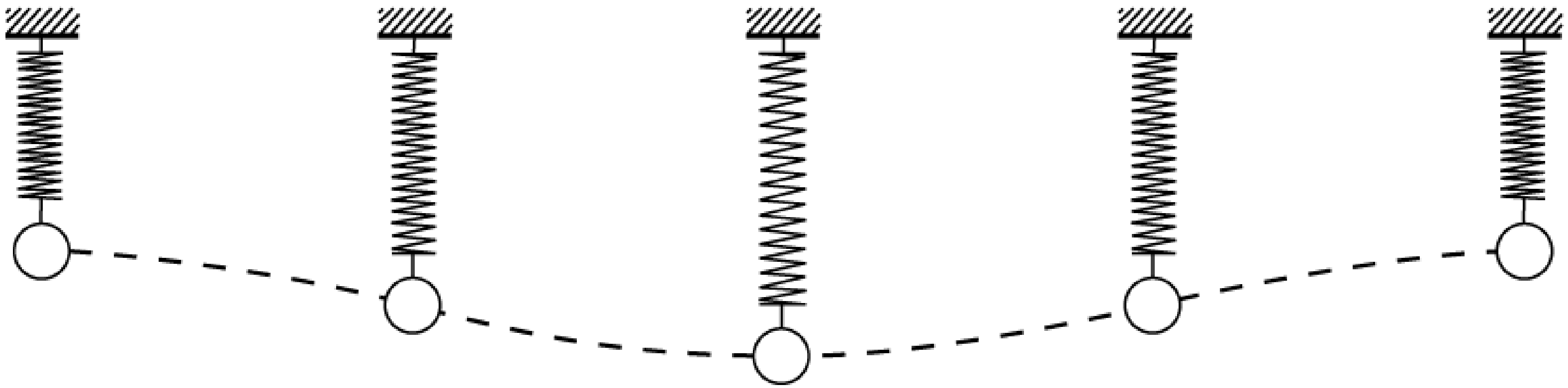
$$\dot{q} = -\omega q_m \sin(\omega t - \varphi);$$

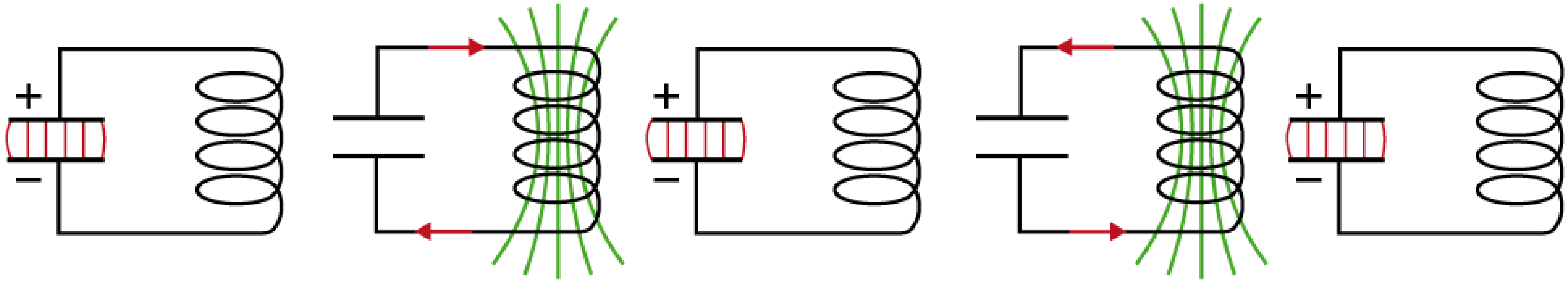
$$\ddot{q} = -\omega^2 q_m \cos(\omega t - \varphi) = -\omega^2 q$$

$$\Rightarrow \quad -L\omega^2 q + \frac{1}{C} q = 0; \quad \text{erfüllt für } \omega^2 = \frac{1}{LC}$$

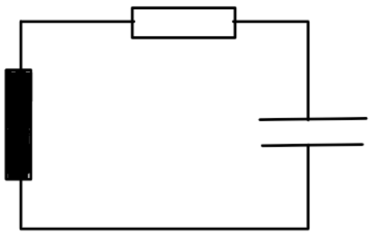
Strom: $$I \doteq \frac{\mathrm{d}q}{\mathrm{d}t} = -\omega \cdot q_m \cdot \sin(\omega t - \varphi)$$

- **Mechanische und elektromagnetische Schwingung**





4.4.2 Gedämpfter LCR-Schwingkreis



$$U_{ind} = -L\dot{I} = RI + \frac{q}{C}$$

$$L\dot{I} + RI + \frac{q}{C} = 0 \; ; \; \Big/ \frac{d}{dt}$$

$$L\ddot{I} + R\dot{I} + \frac{1}{C} I = 0 \; ; \qquad \text{DGL}$$

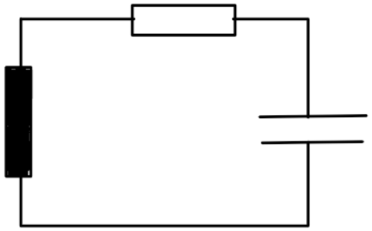
Ansatz: $I = I_0 \, e^{-\gamma t} \cdot e^{i\omega t}$

$$\boxed{I = I_0 \, e^{(i\omega - \gamma)t}}$$

$$\dot{I} = \underbrace{I_0 \, e^{(i\omega-\gamma)t}}_{I} \cdot (i\omega - \gamma)$$

$$\ddot{I} = I \cdot (i\omega - \gamma)^2$$

4.4.2 Gedämpfter LCR-Schwingkreis



in DGL: $$L \cdot I\,(i\omega - \gamma)^2 + R\,I\,(i\omega - \gamma) + \frac{1}{C} \cdot I = 0; \qquad \Big|: I$$

$$L\,(-\omega^2 - 2i\omega\gamma + \gamma^2) + R\,(i\omega - \gamma) + \frac{1}{C} = 0;$$

Imaginärteil: $$-2i\omega\gamma L + i\omega R = 0;$$

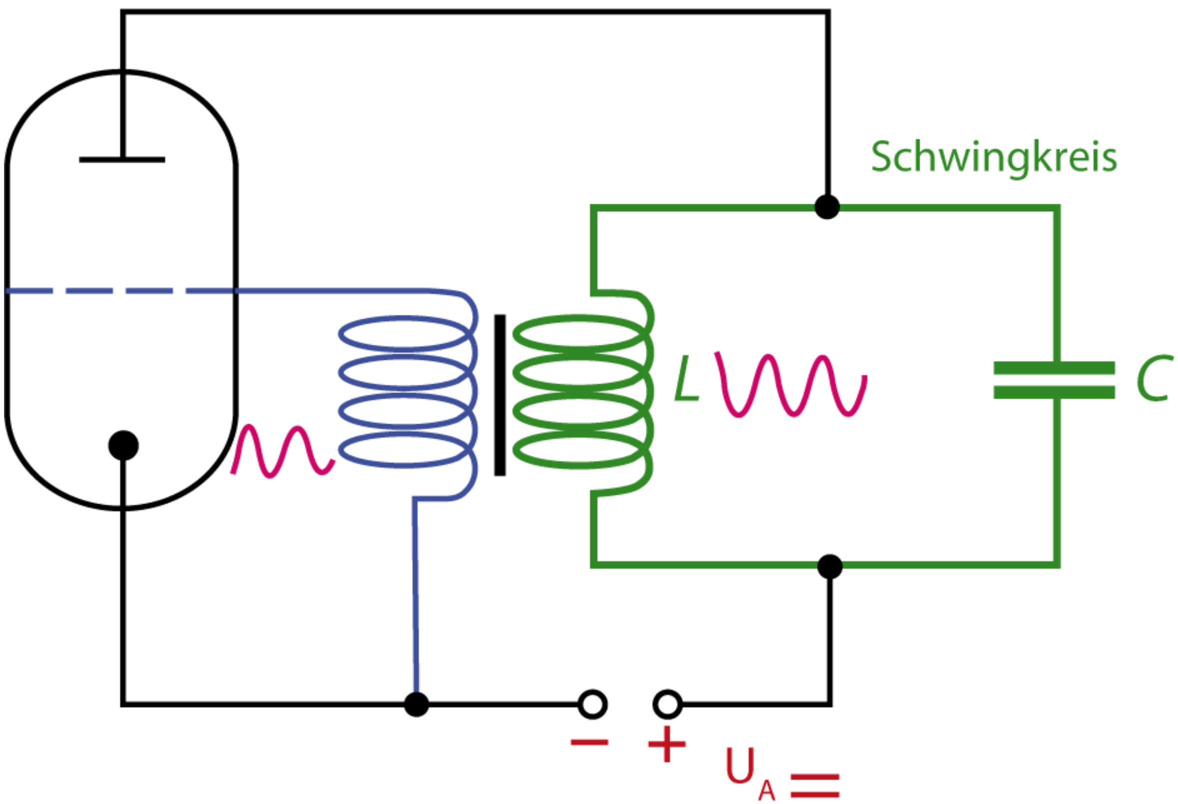
$$\boxed{\gamma = \frac{R}{2L};} \quad (*)$$

Realteil: $$-\omega^2 L + \gamma^2 L - \gamma R + \frac{1}{C} = 0; \qquad \Big|: L$$

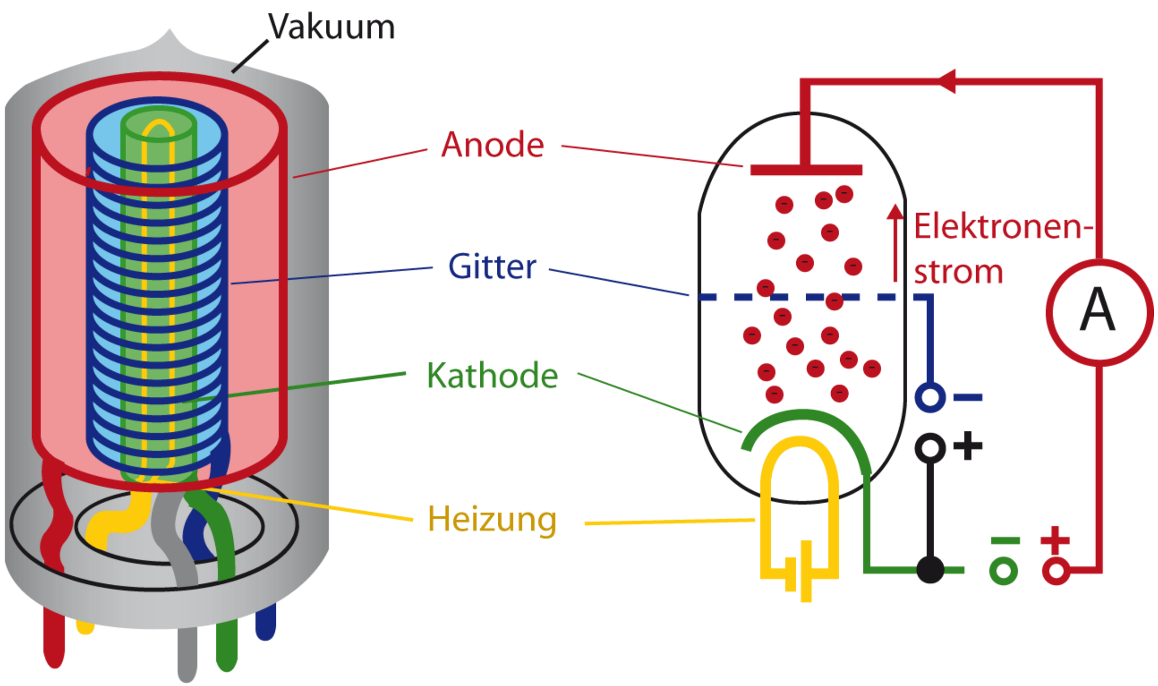
mit (*): $$-\omega^2 + \frac{R^2}{4L^2} - \frac{R^2}{2L} + \frac{1}{LC} = 0;$$

$$\boxed{\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}};}$$

- **Rückkopplungsschaltung nach Meissner**

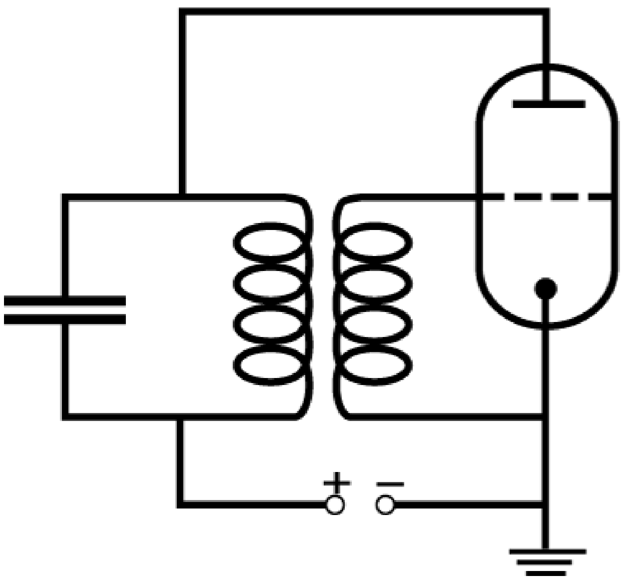


- **Aufbau und Funktionsweise einer Triode**

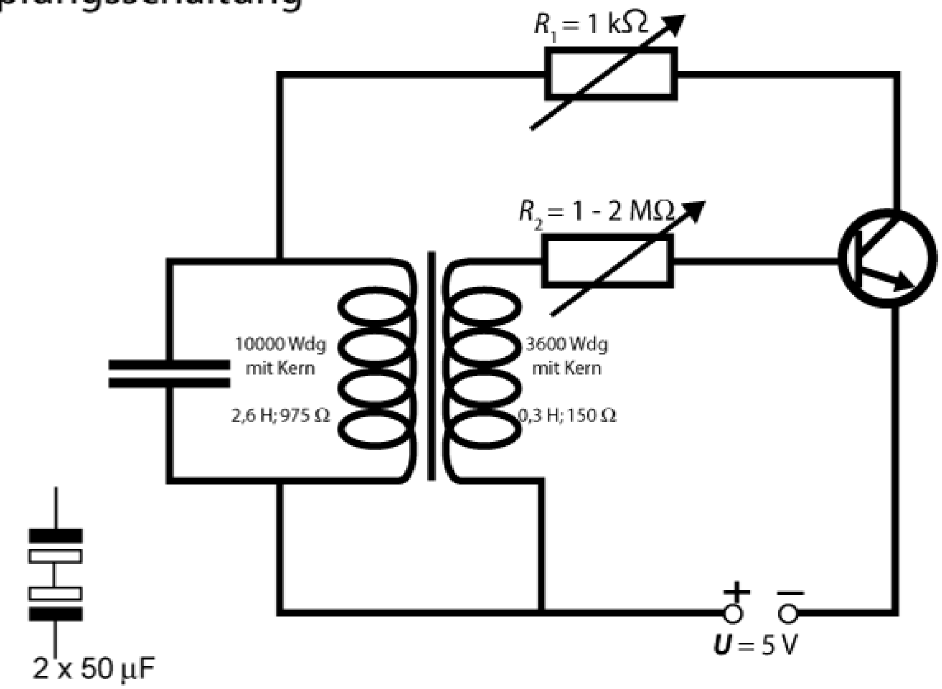


- **Rückkopplungsschaltung nach Meissner**

Meißner- Rückkopplungsschaltung

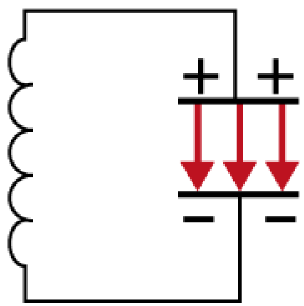


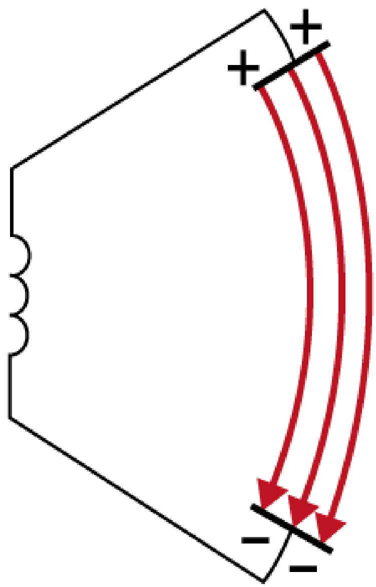
a) gesteuert mit einer Triode

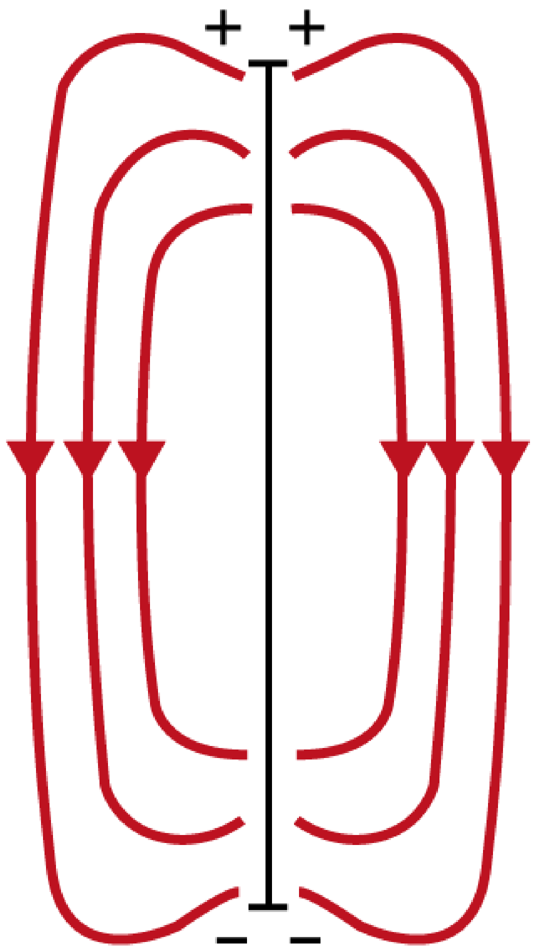


b) gesteuert mit einem Transistor; Beispiel: BC140 Transistor; 2N3022

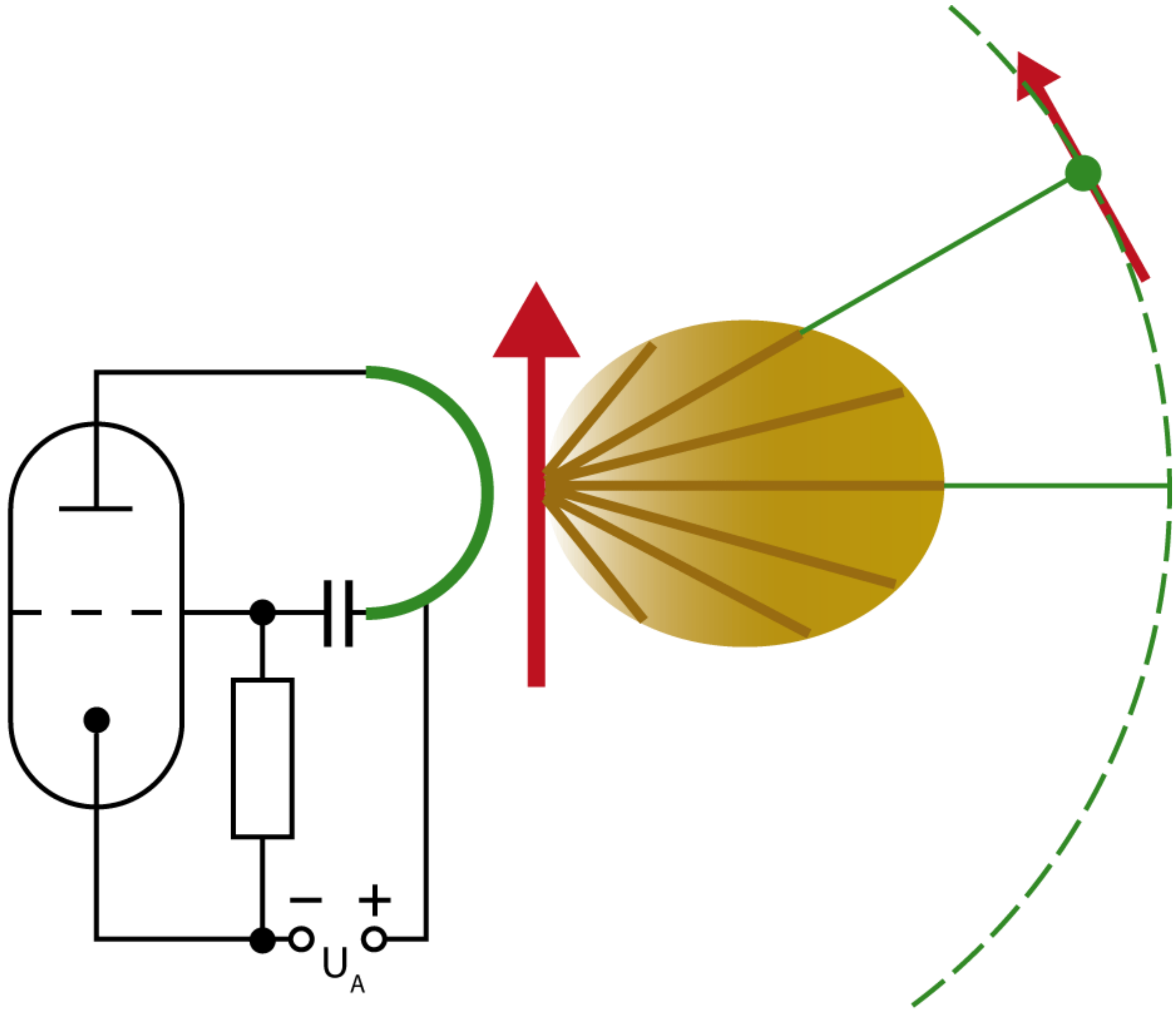
- **Vom LC-Schwingkreis zum Dipolschwinger**







- **Anregung und Abstrahlung eines Dipols**



Schöne vorlesungsfreie Zeit

und schöne Sommerferien